Holographic $1/N_c$ correction from the chiral condensate

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ABSTRACT

We investigate a gravity solution containing the gravitational backreaction of the massive scalar field dual to the chiral condensate, which corresponds to $1/N_c$ correction. In general, condensation changes the vacuum structure, so the present dual geometry is appropriate to describe the chiral condensate vacuum in the gauge theory side. After constructing the dual geometry numerically and applying the hard wall model we study the effect of the $1/N_c$ correction on the lightest meson spectra, which improves the values for lightest meson masses into the observations. In addition, we investigate the chiral condensate dependence the binding energy of heavy quarkonium.

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1 Introduction

Related to the RHIC and LHC experiments there have been many attempts to understand the strongly interacting systems based on the AdS/CFT correspondence [1, 2, 3, 4, 5, 6, 7, 8, 9]. Recently, many holographic models describing the QCD-like gauge theory in the strong coupling regime were invented. Moreover, various techniques used in the holographic QCD are utilized in the holographic condensed matter system, which is called AdS/CMT.

In the framework of a holographic QCD approach, the confinement was realized with an infrared(IR) cut-off in the AdS space, so called the hard wall model [7]. This hard wall model can explain the masses of various mesons, comparing with observations, within 10% error. In Ref. [5] , by using this hard wall model the deconfinement phase transition was also investigated by identifying it with the Hawking-Page transition of the dual gravity theory. There exists a different model, so called soft wall model [8], in which by introducing a non-dynamical scalar field the linear confinement behavior was explained. Usually, we call these two models bottom-up approach. When we consider the string theory origin of the gravity theory, there exist other approaches so called top-down approaches [6], in which after constructing some branes configuration the dual gauge theory is investigated.

In this paper, we will concentrate on the hard wall model. In the original one [7], the $1/N_c$ corrections was not fully considered. There are two different $1/N_c$ corrections. One is coming from the medium effect composed of quarks in the fundamental representation of the gauge group [9, 10]. Following the AdS/CFT correspondence, the gravitational constant $1/2\kappa^2$ is proportional to N_c^2 degrees of freedom and the coupling constant of the dual bulk gauge field, which represents the $SU(N_f)$ flavor group of the dual gauge theory, has $N_c N_f$ degrees of freedom. Since the time-component of the bulk gauge field corresponds to the quark number

density operator in the dual gauge theory, considering the gravitation backreaction of this gauge field maps to investigate the $1/N_c$ corrections coming from the quark density medium. After considering these $1/N_c$ corrections and finding the dual geometries, the Hawking-Page transition, various light meson spectra and the string breaking of the heavy quarkonium were investigated.

There exists the other $1/N_c$ correction coming from the chiral condensate. In [11], the partial corrections of the chiral condensate without the gravitational backreaction were considered. In [12], the backreaction of the massive scalar field with a special type of scalar potential was also investigated. At present paper, the goal is to find a dual geometry corresponding to the chiral condensate vacuum without introducing a special type of scalar potential and investigate various meson spectra. Since the chiral symmetry is restored in the deconfining phase, we concentrate on the confining phase from now on. In the confining phase, the chiral symmetry is spontaneously broken due to the chiral condensate, which was investigated in the holographic model [13]. Usually, this chiral condensate changes the vacuum structure, which implies in the holographic QCD that the dual geometry needs to be deformed. Unlike the gluon condensate background where the analytic solution including the gravitational backreaction of the gluon condensate was found [14, 15], in the chiral condensate background it seems to be impossible to find the dual geometric solution analytically. In this paper, we will calculated the dual geometry numerically and investigate various meson spectra. As will be shown in Sec.4, in the dual geometry including full $1/N_c$ correction coming from the bulk massive scalar field, which represents the chiral condensate and the light quark mass, we obtain the light meson masses very similar to observations.

The rest parts follows: In Sec. 2, we construct the gravity theory dual to chiral condensate vacuum, in which the massive scalar field in the bulk corresponds to the quark mass and the chiral condensate, and find numerical solutions in the chiral limit. Usually, this numerical solution has a naked singularity which causes the IR divergence in the dual QCD, so we introduce a hard wall to avoid this IR divergence. In Sec. 3, on this numerical background we investigate the light meson spectra and string breaking of the heavy quarkonium in the chiral limit. In Sec. 4, we revisit the dual geometry when the light quark mass is not zero. Due to the light quark mass, the asymptotic solution is totally different with one in the chiral limit. After finding the full numerical solution, we investigate the meson spectra depending on the chiral condensate. Especially, when the light quark mass m_q and the chiral condensate σ are 2.383MeV and $(304\text{MeV})^3$ respectively, we obtained a good result explaining the observations. Finally, in Sec. 5 we finish our work with some remarks.

2 Asymptotic AdS background with the chiral condensate

Following the AdS/CFT correspondence, the ground state containing the chiral condensate can be described by the dual gravity theory including the massive scalar field. In this section, to find a dual geometry corresponding to the chiral condensate ground state in QCD we start with

the following action

$$
\begin{aligned}
S &= \int d^5x\sqrt{-G}\left[\frac{1}{2\kappa^2}\left(\mathcal{R}-2\Lambda\right)\right. \\
&\qquad \left. -\mathrm{Tr}\left\{|D\Phi|^2+m^2|\Phi|^2+\frac{1}{4g^2}\left(F^{(L)}_{MN}F^{(L)MN}+F^{(R)}_{MN}F^{(R)MN}\right)\right\}\right], \qquad (1)
\end{aligned}
$$

where $m^2 = -\frac{3}{R^2}$ and the cosmological constant is given by $\Lambda = -\frac{6}{R^2}$. In the above, the superscripts, (L) and (R) in the last term imply the left and right part of $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry group with $F^{(L,R)}_{MN} = \partial_M A^{(L,R)}_N - \partial_N A^{(L,R)}_M - i\left[A^{(L,R)}_M, A^{(L,R)}_N\right]$. In this paper, since we are interested in the chiral condensate ground state we consider bulk gauge fields as fluctuations, which correspond to various vector or axial-vector mesons. In addition, D means a covariant derivative, $D_M\Phi = \partial_M\Phi - iA^{(L)}_M\Phi + i\Phi A^{(R)}_M$. Under the flavor symmetry group, $SU(N_f)_L \times SU(N_f)_R$, the complex scalar field Φ transforms as a bifundamental representation. Now, we set

$$
\Phi(z) = \frac{\phi(z)\mathbf{1}}{2\sqrt{N_f}}\, e^{i\pi^a(z)T^a}, \qquad (2)
$$

where $\mathbf{1}$ means an identity matrix and T^a is a generator of the flavor symmetry group. Furthermore, we identify $\phi(z)$ and $\pi^a(z)$ with the background field and fluctuations respectively, in which the background field ϕ and fluctuation π^a correspond to the quark mass (or chiral condensate) and pseudoscalar mesons.

From (1), the action describing only background fields reduces to

$$
S = \int d^5x\sqrt{-G}\left[\frac{1}{2\kappa^2}\left(\mathcal{R}-2\Lambda\right) - \frac{1}{4}\left\{(\partial\phi)^2 + m^2\phi^2\right\}\right]. \qquad (3)
$$

The equations of motion for the metric and the scalar field $\phi(z)$ are given by

$$
\begin{aligned}
\mathcal{R}_{MN} - \frac{1}{2}G_{MN}\mathcal{R} + G_{MN}\Lambda &= \frac{\kappa^2}{2}\left[\partial_M\phi\partial_N\phi - \frac{1}{2}G_{MN}\left((\partial\phi)^2+m^2\phi^2\right)\right], \\
0 &= \frac{1}{\sqrt{G}}\partial_M\sqrt{G}G^{MN}\partial_N\phi - m^2\phi. \qquad (4)
\end{aligned}
$$

Under the following ansatz

$$
ds^2 = \frac{R^2}{z^2}\left[-F(z)dt^2 + G(z)d\vec{x}^2 + dz^2\right], \qquad (5)
$$

Einstein equation and the equation of motion for the scalar field become

$$
\begin{aligned}
0 &= \frac{F}{4z^2G}\left[\kappa^2G\left(3\phi^2 - z^2\phi'^2\right) - 6z\left(-3G' + zG''\right)\right], \\
0 &= \frac{1}{4}\left[\left(\frac{2F'}{F} - \frac{12}{z}\right)G' - \frac{G'^2}{G} + \left(\kappa^2\phi'^2 + \frac{2zF''-6F'}{zF} - \frac{3\kappa^2\phi^2}{z^2} - \frac{F'^2}{F^2}\right)G + 4G''\right], \\
0 &= \frac{1}{4z^2FG^2}\left[3zGF'\left(zG'-2G\right) - F\left(18zGG' - 3z^2G'^2 + \kappa^2G^2(3\phi^2+z^2\phi'^2)\right)\right], \\
0 &= \frac{1}{4R^2FG}\left[z^2GF'\phi' + F\left(3z^2G'\phi' + 2G(3\phi - 3z\phi' + z^2\phi'')\right)\right], \qquad (6)
\end{aligned}
$$

Here, the solution we want is an asymptotic AdS space, so near the UV boundary ($z = 0$) the unknown functions F and G can be expanded to

$$\begin{aligned} F &= 1 + \sum_{i=1}^{\infty} a_i z^i, \\ G &= 1 + \sum_{i=1}^{\infty} b_i z^i. \end{aligned} \tag{7}$$

Notice that in the pure AdS background a scalar field with $m^2 = -3/R^2$ has the following solution

$$\phi = m_q z + \sigma z^3, \tag{8}$$

where m_q and $\sigma = \langle \bar{q} q \rangle$ are a current quark mass and chiral condensate, respectively. From this fact, when we consider the backreaction of the scalar field, the solution of a massive scalar field near the UV boundary can be expanded as

$$\phi = \sigma z^3 + \sum_{i=4}^{\infty} c_i z^i, \tag{9}$$

where we set $m_q = 0$, which corresponds to the chiral limit. If $m_q \neq 0$, the ansatz in (7) and (9) should be modified to the form containing $\log z$ terms, see Sec. 4. In this section, we concentrate on the chiral limit.

Near the UV boundary, the most general perturbative solutions up to $\mathcal{O}(z^{11})$ are given by

$$\begin{aligned} F &= 1 - 3Mz^4 - \frac{\kappa^2}{12}\sigma^2 z^6 + 4M^2 z^8 + \frac{\kappa^2}{20} M\sigma^2 z^{10}, \\ G &= 1 + Mz^4 - \frac{\kappa^2}{12}\sigma^2 z^6 - \frac{\kappa^2}{60} M\sigma^2 z^{10}, \\ \phi &= \sigma z^3 + \frac{\kappa^2}{16}\sigma^3 z^9, \end{aligned} \tag{10}$$

where new parameter M implies the mass of the black hole. To see this, if turning off σ, the above solutions reduce to the perturbative expansion form of the AdS black hole metric in the Fefferman-Graham coordinate

$$\begin{aligned} F &= \frac{(1 - Mz^4)^2}{1 + Mz^4}, \\ G &= 1 + Mz^4. \end{aligned} \tag{11}$$

In this paper, since we are interested in the meson spectra in the confining phase we set $M = 0$ with a non-zero chiral condensate. Notice that non-zero M corresponds to the Schwarzschild black hole solution dual to the deconfining phase. From (10), the perturbative solution describing the chiral condensate background in the confining phase is given by

$$\begin{aligned} F &= G = 1 - \frac{\kappa^2}{12}\sigma^2 z^6, \\ \phi &= \sigma z^3 + \frac{\kappa^2}{16}\sigma^3 z^9. \end{aligned} \tag{12}$$

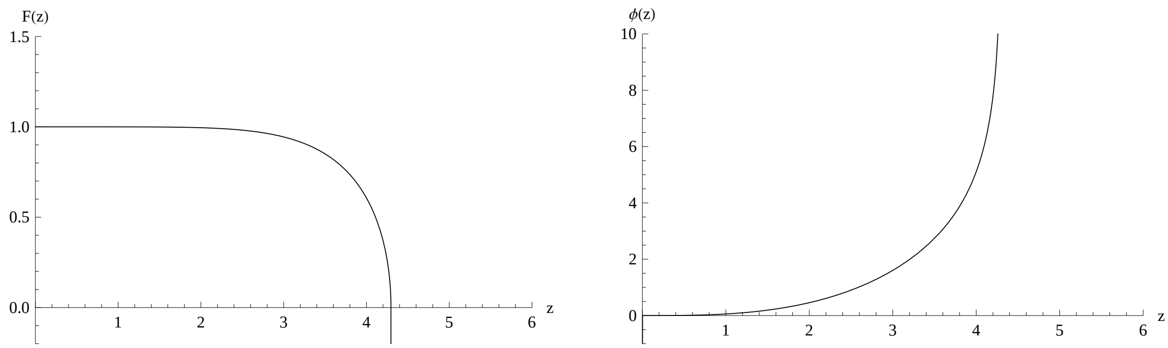

Figure 1: (a) The metric factor $F(z)$ (b) the scalar field $v(z)$ for $\sigma = (0.3846\text{GeV})^3$, in which the metric factor $F(z)$ becomes zero at $z = 4.2963$.

Here since the Lorentz symmetry on the boundary space is restored in the confining phase, F is the exactly same as G. The above in (12) is the solution of

$$\begin{aligned} 0 &= \frac{1}{4z^2}\left[\kappa^2 F\left(3\phi^2 - z^2\phi'^2\right) - 6z\left(zF'' - 3F'\right)\right], \\ 0 &= \frac{1}{4}\left[\frac{6F'^2}{F^2} - \frac{3\kappa^2\phi^2}{z^2} - \frac{24F'}{zF} - \kappa^2\phi'^2\right], \\ 0 &= \frac{1}{2R^2F}\left[2z^2F'\phi' + F\left(3\phi + z(z\phi'' - 3\phi')\right)\right], \end{aligned} \tag{13}$$

which can be obtained from (6) with $F(z) = G(z)$. Note that the first two equations in (6) reduce to the same equation, the first in (13) and that the second corresponds to a constraint equation. Using the boundary behavior of the perturbative solution in (12), we can easily find the full solutions of (13) numerically.

For simplicity, we set $R = 1$ and $N_c = 3$ and use $\frac{1}{\kappa^2} = \frac{\pi^2}{4N_c^2}$ [7]. When the chiral condensate is given by $\sigma = (0.327\text{GeV})^3$, we solve the equations in (13) numerically. As shown in the Figure 1, we find that there exists a naked singularity at $z_s = 4.2963$ (see Figure 2), where the metric factor $F(z)$ becomes zero.

The gravitational backreaction of a bulk massless scalar field corresponding to the gluon condensate was investigated [15]. In the massless scalar case, there exists a geometrical singularity, which can be considered as an IR cut-off. Similarly, can we identify a geometrical singularity in the massive scalar case with an IR cut-off? To answer this question, we should calculate the on-shell gravity action corresponding to the free energy of boundary gauge theory. To do so, we multiply the metric G^{MN} to the first equation in (4). Then, the Einstein equation reduces to

$$\mathcal{R} = \frac{10}{3}\Lambda + \frac{\kappa^2}{2}(\partial\phi)^2 + \frac{5\kappa^2}{6}m^2\phi^2. \tag{14}$$

Inserting this to (3), the on-shell action of this system becomes

$$S_{on} = \int d^5x\sqrt{-G}\left[\frac{2}{3\kappa^2}\Lambda + \frac{1}{6}m^2\phi^2\right]. \tag{15}$$

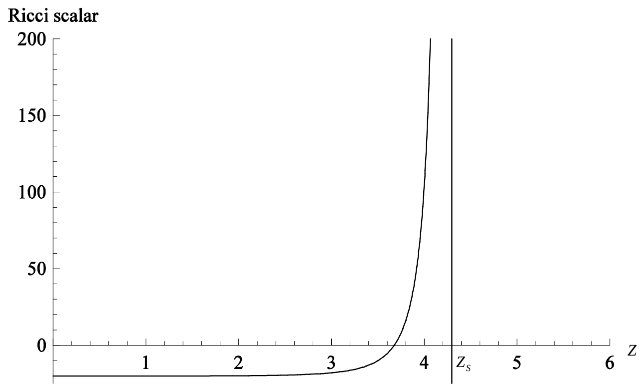

Figure 2: Ricci scalar for $\sigma = (0.3846\text{GeV})^3$, where the vertical line implies the singular point.

In the above, the geometrical singularity makes the on-shell gravity action diverge, so the corresponding boundary free energy suffers from the IR divergence. As a result, it seems to be not appropriate to consider the geometrical singularity as an IR cut-off in the massive scalar case. One way to avoid this problem is to introduce a hard wall as an IR cut-off in front of the geometrical singularity $z_{IR} < z_s$. From now on, we consider this hard wall approach only.

3 Mesons in the chiral limit

3.1 light meson spectra

Following the AdS/CFT correspondence, the bulk gauge field fluctuations correspond to the vector mesons in the holographic QCD. In this section, we will investigate the spectrum of the vector meson by turning on the vector part of the bulk gauge field, which is usually not mixed with the scalar and axial gauge field. In the axial gauge $A_z = 0$, the equation of motion for the vector gauge field $V_i \equiv A_i^{(L)} + A_i^{(R)}$ $(i = 1, 2, 3)$ becomes

$$0 = \frac{1}{\sqrt{G}} \partial_M \sqrt{G} G^{MP} G^{ii} \partial_P V_i, \tag{16}$$

where M, N or μ, ν are the five- or four-dimensional indices, respectively. Using the ansatz,

$$V_\mu = \int \frac{d^4k}{(2\pi)^4} \, e^{-i\omega_n t + i\vec{p}_n \vec{x}} \, V^{(n)}(z), \tag{17}$$

where n implies the n-th excitation mode, the above equation reduces to

$$0 = \partial_z^2 V^{(n)} - \frac{F - zF'}{zF} \partial_z V^{(n)} + \frac{m_n^2}{F} V^{(n)}, \tag{18}$$

where $m_n^2 = w_n^2 - \vec{p}_n^2$ is the mass square of the n-th excited meson state and the prime means a derivative with respect to z. From now on, we will concentrate on the lowest excited state, ρ-meson, and denote its mass by m_ρ.

Here, we have two initial parameters, σ and z_{IR}, so that we should fix these values to evaluate the meson masses. As will be shown, since there is no direct interaction between the

bulk gauge field and the scalar field the vector meson mass does not strongly depend on the chiral condensate. So we can choose $z_{IR} = 1/(0.3227\text{GeV})$, which was used in the Ref. [7] and gave good results for meson masses. Notice that the chiral condensate affects indirectly on the vector meson mass through the change of the metric, though it is very small.

For the axial vector meson $A_\mu \equiv A_\mu^{(L)} - A_\mu^{(R)}$ and pseudoscalar meson π^a, we use the following ansatz

$$\begin{aligned} A_z &= 0, \\ A_\mu^a &= \int d^4k \; e^{iqx} \left(\bar{A}_\mu^a + \partial_\mu \chi^a\right), \\ \pi^a &= \int d^4k \; e^{iqx} \pi^a, \end{aligned} \tag{19}$$

where $\bar{A}$ means a transverse gauge field in the bulk, satisfying $\partial_\mu \bar{A}^\mu = 0$. Then, the equations of motion for them are reduce to

$$\begin{aligned} 0 &= \partial_z \left(\frac{F}{z}\partial_z \bar{A}_\mu\right) + \frac{m_{a_1}^2}{z}\bar{A}_\mu - \frac{g_5^2 F\phi^2}{z^3}\bar{A}_\mu, \\ 0 &= \partial_z \left(\frac{F}{z}\partial_z \chi\right) + \frac{g_5^2 F\phi^2}{z^3}(\pi - \chi), \\ 0 &= -m_\pi^2 \partial_z \chi + \frac{g_5^2 F\phi^2}{z^2}\partial_z \pi, \end{aligned} \tag{20}$$

where m_{a_1} and m_π are masses of the lowest excited mode in the axial vector and pseudoscalar meson, respectively. Moreover, using the holographic recipe [7], the pion decay constant f_π is given by

$$f_\pi^2 = -\left.\frac{1}{g_5^2}\frac{\partial_z \bar{A}(0)}{z}\right|_{z=0}, \tag{21}$$

where $\bar{A}(0)$ is a solution of the first equation in (20) at $m_{a_1} = 0$, satisfying two boundary conditions $\partial_z \bar{A}(z_{IR}) = 0$ and $\bar{A}(0) = 1$.

To see the chiral condensate effect on the meson spectra, we should numerically solve (18) and (20) with appropriate boundary conditions, Neumann and Dirichlet boundary condition at $z = z_{IR}$ and $z = 0$ respectively. Before doing that, we should notice that if σ is greater than $\sigma_c = (0.5337\text{GeV})^3$, the singular point z_s is smaller than z_{IR}. So the on-shell gravity action diverges, which implies that there exists an upper bound of the chiral condensate value. From now on, we consider the case $\sigma < \sigma_c$ only. In the Table 1, we numerically calculate the first-excited meson masses depending on the chiral condensate and we plot these data in Figure 3. As the chiral condensate decreases, the vector meson mass grows slightly but the axial-vector meson mass, the pion mass, and the pion decay constant decrease. Moreover, the masses of the vector and axial-vector meson have the same value at $\sigma = 0$, which is due to the restoration of the flavor symmetry. In addition, since the pion mass is sufficiently small the Gell-Mann-Oakes-Renner (GOR) relation is 'weakly' satisfied up to 10^{-12}GeV^4 order in the chiral limit $m_q = 0$

$$f_\pi^2 m_\pi^2 \approx 2 m_q \sigma. \tag{22}$$

σ (GeV^3)	m_ρ (GeV)	m_{a_1} (GeV)	m_π (GeV)	f_π(GeV)	Δ (GeV^4)
0	0.7760	0.7760			
$(0.100)^3$	0.7760	0.7769	2.95×10^{-6}	4.79×10^{-3}	1.99×10^{-16}
$(0.200)^3$	0.7760	0.8304	3.15×10^{-6}	3.59×10^{-2}	1.28×10^{-14}
$(0.250)^3$	0.7760	0.9598	3.66×10^{-6}	6.03×10^{-2}	4.88×10^{-14}
$(0.304)^3$	0.7758	1.2217	4.78×10^{-6}	8.31×10^{-2}	1.58×10^{-13}
$(0.350)^3$	0.7755	1.4754	6.18×10^{-6}	9.81×10^{-2}	3.68×10^{-13}
$(0.385)^3$	0.7751	1.6442	7.46×10^{-6}	10.82×10^{-2}	6.51×10^{-13}
$(0.400)^3$	0.7748	1.7115	8.04×10^{-6}	11.25×10^{-2}	8.19×10^{-13}
$(0.450)^3$	0.7735	1.9281	10.18×10^{-6}	12.66×10^{-2}	1.66×10^{-12}
$(0.500)^3$	0.7709	2.1424	12.57×10^{-6}	14.07×10^{-2}	3.12×10^{-12}

Table 1: Various meson masses depending on the chiral condensate. Here, the subscripts in the first line, ρ, a_1 and π, imply the first excited modes of the vector, axial vector and pseudoscalar mesons.

In Figure 4, we plot the value of $\Delta = f_\pi^2 m_\pi^2$ and the chiral condensate dependence of the pion decay constant.

3.2 binding energy of the heavy quarkonium

In this section, we will investigate the binding energy of a heavy quarkonium depending on the chiral condensate. First, we consider a heavy quarkonium composed of two heavy quarks on the chiral condensate background obtained by numerical calculation in the previous section. Then, the action for a fundamental string connecting two heavy quarks is given by

$$S = \frac{1}{2\pi\alpha'} \int d^2\sigma \, \sqrt{\det h_{\alpha\beta}}, \tag{23}$$

where $h_{\alpha\beta}$ is an induced metric on the string worldsheet. For simplicity, we take $R = 1$ and $\alpha' = 1/2\pi$.

Under the following gauge fixing

$$\tau = t, \quad \sigma = x^1 \equiv x \quad \text{and} \quad z = z(x), \tag{24}$$

the fundamental string action on the background metric (5) with $F = G$ reduces to

$$S = \int_{-T/2}^{T/2} dt \int_{-r/2}^{r/2} dx \, \frac{\sqrt{F^2 + F z'^2}}{z^2}, \tag{25}$$

where r is the distance between quark and anti-quark in the x^1 direction. The conserved Hamiltonian of this system is

$$H = -\frac{F^2}{z^2 \sqrt{F^2 + F z'^2}}. \tag{26}$$

If there exists z_0 where $\frac{\partial z}{\partial x}\big|_{z=z_0}$ becomes zero, the Hamiltonian at this point is given by

$$H = -\frac{F_0}{z_0^2}, \tag{27}$$

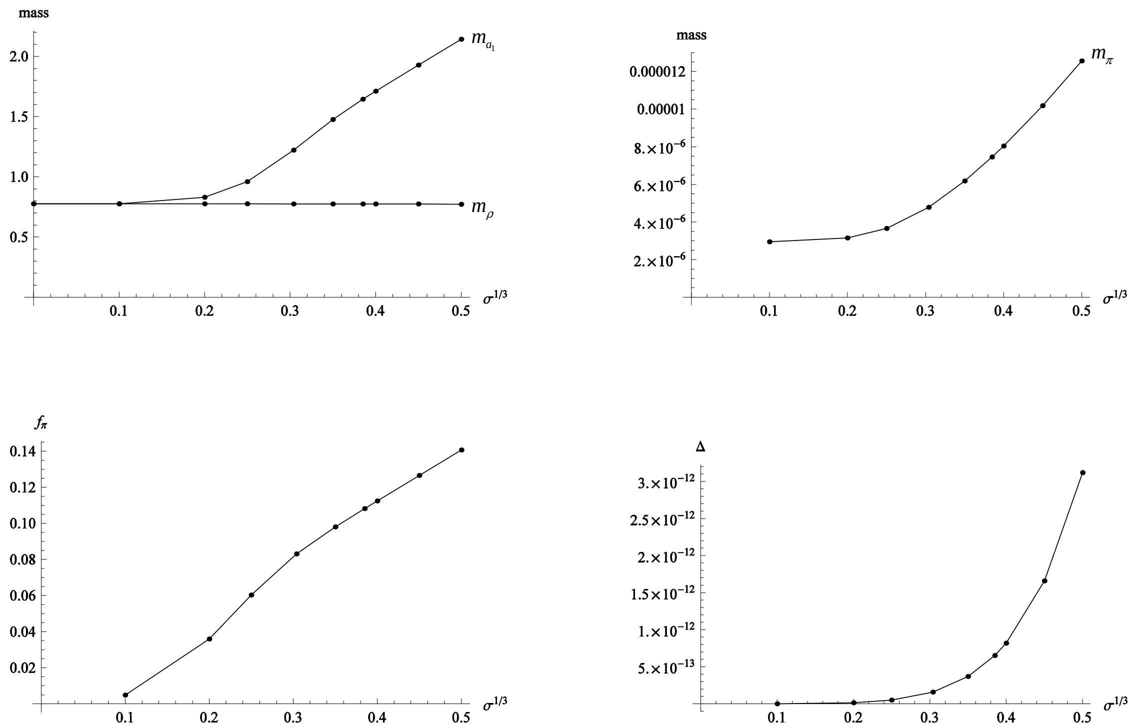

Figure 3: Chiral condensate dependence of vector-, axial vector-mesons, pion and the pion decay constant. In the right-below figure, we plot the deviation from GOR relation, in which we found that GOR relation is almost satisfied in the chiral limit.

where F_0 is the value of F at $z = z_0$. Due to the conservation law, we can find the integral relation between r and z_0 from two equations, (26) and (27)

$$r = 2 \int_0^{z_0} dz \, \frac{z^2 F_0}{\sqrt{F \left(z_0^4 F^2 - z^4 F_0^2 \right)}}, \tag{28}$$

and the energy of this string configuration

$$E = 2 \int_0^{z_0} dz \, \frac{z_0^2 F^{3/2}}{z^2 \sqrt{z_0^4 F^2 - z^4 F_0^2}}. \tag{29}$$

Notice that since we are considering the static string configuration the kinetic energy of two heavy quarks can be ignored. Then, the above energy corresponds to the potential energy between two heavy quarks including the heavy quark masses. Notice that the above potential energy diverges because in the hard wall model the heavy quark have infinite mass. To consider the interaction energy only, we should subtract this infinite mass term described by the straight string configuration. To do so, we consider two straight strings describing the infinite mass of two quarks with the following ansatz

$$\tau = t \quad \text{and} \quad \sigma = z. \tag{30}$$

Then, the energy of two straight strings become

$$2M = 2 \int_0^{z_{IR}} dz \, \frac{\sqrt{F}}{z^2}, \tag{31}$$

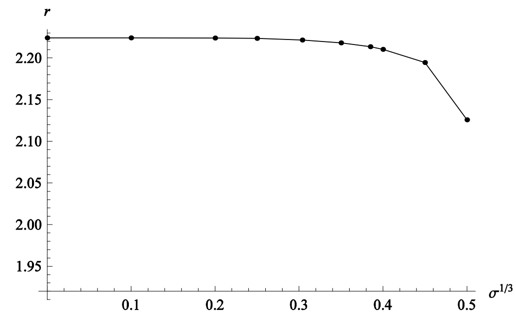

Figure 4: In the chiral limit $m_q = 0$, the string breaking distance depending on the chiral condensate.

which diverges and can be interpreted as masses of two heavy quarks. So the interaction energy of heavy quarkonium V is finally given by

$$V = 2\int_0^{z_0} dz \, \frac{z_0^2 F^{3/2}}{z^2\sqrt{z_0^4 F^2 - z^4 F_0^2}} - 2\int_0^{z_{IR}} dz \, \frac{\sqrt{F}}{z^2}, \tag{32}$$

which is finite. If the interaction energy between two heavy quarks is larger than the mass of two light quarks, the heavy quarkonium can break into two heavy-light meson, which is a bound state of the heavy and light quarks. Usually, this phenomena is called the string breaking. Here, since we assume that the mass of the light quark is zero, the string breaking occurs when the interaction energy becomes zero. In Figure 4, we plot the string breaking distance, at which the heavy quarkonium breaks to two heavy-light mesons. As the chiral condensate increases the string breaking distance becomes short, which implies that it is easy to break the heavy quarkonium to two heavy-light meson at the small chiral condensate regime.

4 Mesons with non-zero m_q

4.1 light meson spectra

In this section, we improve the hard wall model to the case with non-zero quark mass. As mentioned previously, the metric solution of (4) can be described by a most general ansatz

$$ds^2 = \frac{R^2}{z^2}\left[-F(z)dt^2 + G(z)d\vec{x}^2 + dz^2\right]. \tag{33}$$

This ansatz corresponds to the scalar field deformation of the black hole solution in the Fefferman-Graham coordinate, see an example (10). So, this ansatz is proper to describe the black hole geometry dual to the deconfinement phase. For the confining phase, the dual geometry should be described by a deformation of the AdS type metric, where the Lorentz symmetry of the boundary space is restored. Therefore, the good ansatz for the dual geometry of the confining phase is

$$ds^2 = \frac{R^2}{z^2}\left[F(z)\left(-dt^2 + d\vec{x}^2\right) + dz^2\right]. \tag{34}$$

Since our interest is to investigate the meson spectra in the confining phase including the chiral condensate, we concentrate on the latter case. Then, the equations of motion for this system becomes (13). Note that to find the perturbative solutions of (13), the appropriate log terms should

be added to the ansatz for metric and the scalar field. With those log terms, the perturbative solutions near the boundary are given by

$$
\begin{aligned}
F(z) &= 1-\frac{\kappa^2}{12}m_q^2z^2+\frac{1}{144}\Big(\kappa^4m_q^4-18\kappa^2m_q\sigma-3\kappa^4m_q^4\log z\Big)z^4 \\
&\quad+\frac{1}{31104}\Big(65\kappa^6m_q^6-396\kappa^4m_q^3\sigma-2592\kappa^2\sigma^2-66\kappa^6m_q^6\log z \\
&\quad-864\kappa^4m_q^3\sigma\log z-72\kappa^6m_q^6\log z^2\Big)z^6+\cdots
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
\phi(z) &= m_qz+\Big(\sigma+\frac{1}{6}\kappa^2m_q^3\log z\Big)z^3+\frac{1}{576}\Big(-13\kappa^4m_q^5+144\kappa^2m_q^2\sigma+24\kappa^4m_q^5\log z\Big)z^5 \\
&\quad+\frac{1}{62208}\Big(-209\kappa^6m_q^7+1044\kappa^4m_q^4\sigma+10368\kappa^2m_q\sigma^2+174\kappa^6m_q^7\log z \\
&\qquad+3456\kappa^4m_q^4\sigma\log z+288\kappa^6m_q^7\log z^2\Big)z^7 \\
&\quad+\frac{1}{23887872}\Big(-2171\kappa^8m_q^9-165744\kappa^6m_q^6\sigma+1586304\kappa^4m_q^3\sigma^2+1492992\kappa^2\sigma^3 \\
&\qquad-27624\kappa^8m_q^9\log z+528768\kappa^6m_q^6\sigma\log z+746496\kappa^4m_q^3\sigma^2\log z \\
&\qquad+44064\kappa^8m_q^9\log z^2+124416\kappa^6m_q^6\sigma\log z^2+6912\kappa^8m_q^9\log z^3\Big)z^9 \\
&\quad+\cdots.
\end{aligned}
\tag{36}
$$

This solution reduces to (12) when $m_q=0$. Using these as initial data for solving the equations of motion, we can find various meson spectra depending on the chiral condensate numerically, see Table 2 and Figure 5. Comparing Table 2 with Table 1, we can see that though the masses of the vector and axial-vector meson do not crucially depend on the current quark mass, the magnitude of the current quark mass is very important to determine the pion mass and pion decay constant. Here, for more realistic data, we choose $z_{IR}=1/(0.3227\text{GeV})$ and $m_q=0.002383\text{GeV}$. In this case, which give good data similar to the observed meson spectra. In the improved EKSS model including the $1/N_c$ correction of the chiral condensate, the masses of ρ-, a_1-meson and pion at $\sigma=(304\text{MeV})^3$ are almost the same as the experimental data (see Table 2). In Table 3, we compare our results with experimental observations [16] and those in the EKSS B-model [7]. Furthermore, we plot data of Table 2 in Figure 5.

There are several remarkable points.

1. Notice that although there is no direct coupling between the vector meson and the massive scalar field in this model, due to the gravitational backreaction of the massive scalar field the ρ-meson mass depends on the massive scalar field indirectly. From this fact, we can expect that the ρ-meson mass does not significantly depend on the massive scalar field corresponding the chiral condensate. In Figure 3 and Figure 5, we can see that increasing of the chiral condensate does not significantly change the ρ-meson mass. Instead, the ρ-meson mass grows down slightly as the chiral condensate increases.

2. When we turn on the quark mass and chiral condensate, the improved EKSS model including

σ (GeV^3)	m_ρ (GeV)	m_{a_1} (GeV)	m_π (GeV)	f_π (GeV)	Δ (GeV^4)
0	0.77604	0.7762	0.7762	0.0084	4.30×10^{-5}
$(0.100)^3$	0.77604	0.7777	0.3946	0.0108	1.35×10^{-5}
$(0.150)^3$	0.77604	0.7884	0.2400	0.0200	6.99×10^{-6}
$(0.200)^3$	0.77602	0.8354	0.1698	0.0388	5.19×10^{-6}
$(0.250)^3$	0.77596	0.9675	0.1427	0.0624	4.97×10^{-6}
$(0.304)^3$	0.77580	1.2306	0.1396	0.0846	5.75×10^{-6}
$(0.350)^3$	0.77549	1.4834	0.1462	0.0995	7.06×10^{-6}
$(0.385)^3$	0.77506	1.6517	0.1528	0.1096	8.32×10^{-6}
$(0.400)^3$	0.77480	1.7189	0.1556	0.1138	8.89×10^{-6}
$(0.450)^3$	0.77345	1.9354	0.1650	0.1279	1.10×10^{-5}
$(0.500)^3$	0.77083	2.1497	0.1739	0.1419	1.34×10^{-5}

Table 2: meson masses with the chiral condensate in the hard wall approach. ($\Delta = f_\pi^2 m_\pi^2 - 2 m_q \sigma$)

the chiral condensate gives good results consistent with the measured values (see the case with $m_q = 2.383$MeV and $\sigma = (304\text{MeV})^3$ in Table 2 and Table 3).

3. In the chiral limit, because the Lorentz symmetry of the boundary thoery is fully restored at $\sigma = 0$, ρ- and a_1-meson has the same mass, which was also studied in Ref. [17] without considering the gravitational backreaction of the scalar field. For the non-zero quark mass case, the chiral symmetry is almost restored in the limit $\sigma \to 0$ although the explicit quark mass breaks this chiral symmetry slightly. So, at $\sigma = 0$ in Table 2, the ρ-meson mass is slightly different with those of a_1-meson and pion.

4. In Figure 5 and Table 2, there exists a critical value of the chiral condensate $\sigma \approx (0.26\text{GeV})^3$. As the chiral condensate decreases, the pion mass grows down above this critical point as we expect, while below it the pion mass grows. This non-trivial chiral condensate dependence of the pion mass can be understood in the gravity side by the flavor symmetry restoration and the mixing between the pion and the longitudinal mode of the axial vector. Due to the flavor symmetry restoration at the small σ region, all vector and axial vector mesons have the similar mass. Furthermore, for $m_q \neq 0$ the mixing between the pion and the longitudinal mode of the axial vector make the pion mass have the similar mass. In the chiral limit as shown in Figure3, there is no such chiral condensate dependence of the pion mass. In this case although the flavor symmetry is fully restored the mixing between the pion and the longitudinal mode of the axial vector disappears because ϕ becomes zero when $\sigma \to 0$. This fact implies that the small current quark mass plays an important role to determine the pion mass in the small chiral condensate regime. Though we do not have good understanding about the meaning of this behavior in the dual QCD side, since the ratio between the pion mass and the chiral condensate $m_\pi/\sigma^{1/3}$ is larger than 1 below the critical chiral condensate, the usual chiral perturbation method in QCD does not work in this regime. As a result, the results obtained here would shed light on understanding physics, which can not be described by the chiral perturbation theory.

	Measured [16]	EKSS B-model [7]	chiral condensate background
z_{IR}		1/(346 MeV)	1/(322.7 MeV)
m_q		2.3 MeV	2.383 MeV
σ		$(308\text{ MeV})^3$	$(304\text{ MeV})^3$
m_ρ	775.8 ± 0.5 MeV	832 MeV	775.8 MeV
m_{a_1}	1230 ± 40 MeV	1220 MeV	1230.6 MeV
m_π	139.6 ± 0.0004 MeV	141 MeV	139.6 MeV
f_π	92.4 ± 0.35 MeV	84.0 MeV	84.6 MeV

Table 3: meson masses without and with the chiral condensate.

Finally, to confirm and understand numerical meson spectra in the $\sigma \to 0$ limit, we will investigate (18) and (20) analytically in the small σ case. Notice that when the chiral condensate is very small, $F(z)$ and $\phi(z)$ can be approximated as 1 and $m_q z$ in the range of $0 \leq z \leq z_{IR}$. Using this, the solution of the last equation in (20) becomes

$$\pi(z) = \frac{m_\pi^2}{2\pi m_q^2}\chi(z) + c, \tag{37}$$

where c is a constant. From the boundary conditions, $\pi(0) = \chi(0) = 0$, c should be fixed as 0. Inserting this result into the second equation in (20), we finally obtain

$$0 = \partial_z\left(\frac{1}{z}\partial_z\chi\right) + \frac{m_\pi^2 - g_5^2 m_q^2}{z}\chi. \tag{38}$$

From this, we can see that in the small chiral condensate limit with very small quark mass, the pion mass is proportional to that of the longitudinal mode of the axial vector meson χ, which makes the pion mass grow as the chiral condensate goes to zero. In the $\sigma \to 0$ limit, the other equations for the vector and axial vector meson can be rewritten as

$$\begin{aligned} 0 &= \partial_z\left(\frac{1}{z}\partial_z V^{(n)}\right) + \frac{m_\rho^2}{z}V^{(n)}, \\ 0 &= \partial_z\left(\frac{1}{z}\partial_z \bar{A}_\mu\right) + \frac{m_{a_1}^2 - g_5^2 m_q^2}{z}\bar{A}_\mu. \end{aligned} \tag{39}$$

Therefore, we can see that the transverse mode of the axial vector $\bar{A}_\mu$ and pion π have the same mass at $\sigma = 0$, which was shown in the Table 2. Moreover, from the above we can find a mass relation between the vector meson and pion

$$m_\rho^2 \approx m_\pi^2 - g_5^2 m_q^2, \tag{40}$$

which is not the exact relation but the approximation. So, in the case $m_q \ll m_\pi$, we can see that the vector, axial vector meson and pion have the similar mass in the case $\sigma = 0$, which gives consistent results with Table 2. In the chiral limit, as shown in the previous section, the pion mass becomes very small as $\sigma \to 0$. Though the chiral symmetry is restored for $\sigma = 0$, the

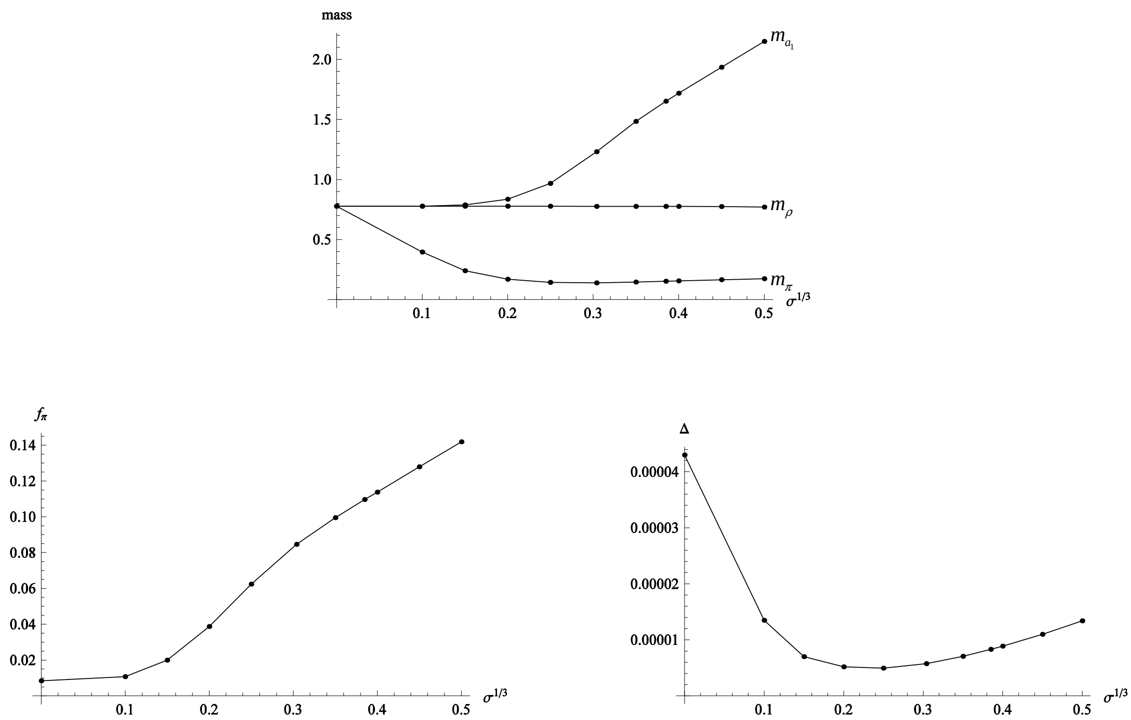

Figure 5: For $m_q = 2.383$MeV, we plot the masses of vector, axial vector meson and pion (the upper figure) and the pion decay constant (left below). In the right below figure, the deviation from GOR relation is plotted. For small chiral condensate regime, it seems that the GOR relation does not work.

pion and the longitudinal mode of the axial vector do not mix because the scalar field is zero in the chiral limit. So we can see that the existence of the light quark mass significantly affects on the pion mass, see Table 1 and 2.

4.2 binding energy of the heavy quarkonium

In the previous section, we have investigated the string breaking of the heavy quarkonium when $m_q = 0$. In this section, we will study the string breaking for $m_q \neq 0$. Since the method to calculate the interaction energy of the heavy quarkonium is the same as the previous section, we summarize the results with some comments.

For $m_q \neq 0$, the interaction energy of the heavy quarkonium is given by the same form as (32) with F defined in (35). As mentioned, for the string breaking the interaction energy should be greater than masses of two light quarks. In this case, since the light quark has mass $m_q = 2.383$MeV the string breaking occurs at $V = 2m_q$.

As we expect, since the quark mass is too small the string breaking distance is not significantly changed from one obtained in the chiral limit, see Table 4. Anyway, the non-zero quark mass make the string breaking distance large for the small chiral condensate regime, roughly $\sigma \lesssim (0.45\text{GeV})^3$. This implies that the non-zero light quark mass makes the string breaking slightly more difficult.

$\sigma(\mathrm{GeV}^3)$	distance for $m_q = 0$ (GeV^{-1})	distance for $m_q \neq 0$ (GeV^{-1})
$(0.000)^3$	2.22426	2.25761
$(0.100)^3$	2.22426	2.25760
$(0.200)^3$	2.22406	2.25739
$(0.250)^3$	2.22348	2.25681
$(0.304)^3$	2.22171	2.25503
$(0.350)^3$	2.21825	2.25155
$(0.385)^3$	2.21342	2.24670
$(0.400)^3$	2.21048	2.24375
$(0.450)^3$	2.19467	2.22782
$(0.500)^3$	2.12580	2.05779

Table 4: String breaking distance depending on the chiral condensate in cases, $m_q = 0$ and $m_q \neq 0$.

5 Discussion

We have studied $1/N_c$ correction of the meson spectra, which comes from the light quark mass and/or chiral condensate. To encoding the chiral condensate effect to the dual gravity theory, the gravitational backreaction of the massive scalar field, which corresponds to light quark and/or the chiral condensate, was considered. From the initial data determined by the asymptotic solutions, we found the numerical solutions for metric and scalar field, in which there exists a geometrical singularity. Because this singularity causes the IR divergence of the dual gauge theory, we introduce an IR cut-off to avoid it.

In this hard wall model, both the light meson spectra and heavy quarkonium binding energy depending on the chiral condensate have been investigated. For masses of the vector and axial vector meson, the chiral condensate dependence is almost same in both cases $m_q = 0$ and $m_q \neq 0$. In other words, as the chiral condensate increases, the vector meson mass decreases slightly while the axial vector meson mass increases significantly. In the $\sigma \to 0$ limit, since the chiral symmetry is fully restored the masses of the vector and axial vector have the same value in the chiral limit. In the case of $m_q \neq 0$, since the explicit light quark mass breaks the chiral symmetry slightly, there exists small difference between the vector and axial vector meson masses. As a result, the vector and axial vector meson masses do not significantly depend on the light quark mass, while the pion mass depends on the light quark mass significantly. In the chiral limit, due to the absence of the quark mass the pion mass is very small compared with the observation. This implies that though light quark mass is very small it plays a crucial role to determine the pion mass. So we have recalculated the pion mass after encoding the light quark mass to the dual background. Especially, we found that due to this $1/N_c$ correction, in the case of $m_q = 2.383$ MeV and $\sigma = (304\mathrm{MeV})^3$, the original results obtained in [7] are improved to the real observations.

In the chiral limit, the pion mass increases as the chiral condensate increases, which looks qualitatively consistent with the chiral perturbation theory in QCD. However, in the case $m_q \neq 0$ there exists a critical value of the chiral condensate $\sigma_c \approx (0.26\text{GeV})^3$. As the chiral condensate grows, the pion mass decreases (increases) below (above) this critical value. In the $\sigma \to 0$ limit for $m_q \neq 0$, the pion and axial vector meson masses have the same value due to the mixing of them as well as the restoration of the chiral symmetry. In the chiral limit, though the chiral symmetry is fully restored, there is no mixing between pion and axial vector meson in the $\sigma \to 0$ limit, so that pion can have very small mass compared with the axial vector meson mass. This unexpected behavior of the pion mass for $m_q \neq 0$ can be well understood in the gravity side but in the gauge theory side it is not clear why this behavior occurs. The existence of the critical chiral condensate seems to indicate that the chiral perturbation theory in QCD is only applicable above this critical value and below it new approach is needed. So it would be very interesting to understand, in the gauge theory side, the unexpected behavior of the pion mass below the critical chiral condensate.

We also observed the heavy quarkonium binding energy from the string breaking distance for zero and non-zero quark masses. On both cases, we found that the string breaking distance becomes short as the chiral condensate increases. This implies that the heavy quarkonium is broken to two heavy-light mesons more easily at the large chiral condensate regime. We also investigated the string breaking depending on the light quark mass. As we can expected, we found that since the light quark mass is too smaller than the heavy quarkonium mass, the string breaking of the heavy quarkonium does not crucially depend on the light quark mass.

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References

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109]; E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150]; O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri

and Y. Oz, Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111]; J. Erdmenger, N. Evans, I. Kirsch and E. Threlfall, Eur. Phys. J. A **35**, 81 (2008) [arXiv:0711.4467 [hep-th]].

[2] J. Erdmenger, N. Evans, I. Kirsch and E. Threlfall, Eur. Phys. J. A **35**, 81 (2008) [arXiv:0711.4467 [hep-th]].

[3] U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti, JHEP **0905**, 033 (2009) [arXiv:0812.0792 [hep-th]].

[4] G. Policastro, D. T. Son and A. O. Starinets, JHEP **0209**, 043 (2002) [arXiv:hep-th/0205052]; G. Policastro, D. T. Son and A. O. Starinets, JHEP **0212**, 054 (2002) [arXiv:hep-th/0210220].

[5] C. P. Herzog, Phys. Rev. Lett. **98**, 091601 (2007) [arXiv:hep-th/0608151].

[6] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141]; T. Sakai and S. Sugimoto, Prog. Theor. Phys. **114**, 1083 (2005) [arXiv:hep-th/0507073].

[7] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005) [arXiv:hep-ph/0501128].

[8] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006) [arXiv:hep-ph/0602229].

[9] L. Da Rold and A. Pomarol, Nucl. Phys. B **721**, 79 (2005) [arXiv:hep-ph/0501218].

[10] B. H. Lee, C. Park and S. J. Sin, JHEP **0907**, 087 (2009) [arXiv:0905.2800 [hep-th]]; C. Park, Phys. Rev. D **81**, 045009 (2010) [arXiv:0907.0064 [hep-ph]]; K. Jo, B. H. Lee, C. Park and S. J. Sin, JHEP **1006**, 022 (2010) [arXiv:0909.3914 [hep-ph]]; S. J. Sin, JHEP **0710**, 078 (2007) [arXiv:0707.2719 [hep-th]].

[11] Y. Kim, B. H. Lee, S. Nam, C. Park and S. J. Sin, Phys. Rev. D **76**, 086003 (2007) [arXiv:0706.2525 [hep-ph]].

[12] J. P. Shock, F. Wu, Y. L. Wu and Z. F. Xie, JHEP **0703**, 064 (2007) [arXiv:hep-ph/0611227]; Y. L. Wu and Z. F. Xie, JHEP **0710**, 009 (2007) [arXiv:0705.2360 [hep-ph]].

[13] M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, JHEP **0405**, 041 (2004) [arXiv:hep-th/0311270]; N. J. Evans and J. P. Shock, Phys. Rev. D **70**, 046002 (2004) [arXiv:hep-th/0403279].

[14] S. S. Gubser, arXiv:hep-th/9902155; A. Kehagias and K. Sfetsos, Phys. Lett. B **454**, 270 (1999) [arXiv:hep-th/9902125]; C. Csaki, J. Erlich, C. Grojean and T. J. Hollowood, Nucl. Phys. B **584**, 359 (2000) [arXiv:hep-th/0004133]; S. Nojiri and S. D. Odintsov, Phys. Rev.

D **61**, 024027 (2000) [arXiv:hep-th/9906216]; D. Bak, M. Gutperle, S. Hirano and N. Ohta, Phys. Rev. D **70**, 086004 (2004) [arXiv:hep-th/0403249]; C. Csaki and M. Reece, JHEP **0705**, 062 (2007) [arXiv:hep-ph/0608266]; Y. Kim, B. H. Lee, C. Park and S. J. Sin, JHEP **0709**, 105 (2007) [arXiv:hep-th/0702131]; Y. Kim, B. H. Lee, C. Park and S. J. Sin, Phys. Rev. D **80**, 105016 (2009) [arXiv:0808.1143 [hep-th]].

[15] Y. Ko, B. H. Lee and C. Park, JHEP **1004**, 037 (2010) [arXiv:0912.5274 [hep-th]].

[16] S. Eidelman *et al.* [Particle Data Group], Phys. Lett. B **592**, 1 (2004).

[17] Y. Kim and H. K. Lee, Phys. Rev. D **77**, 096011 (2008) [arXiv:0802.2409 [hep-ph]].